Deviance and Vice: strength as a theoretical virtue in the epistemology of logic*

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(Rutgers Epistemology Conference 2017)

November 1, 2017

Abstract

This paper is about the putative theoretical virtue of strength, as it might be used in abductive arguments to the correct logic in the epistemology of logic. It argues for three theses. The first is that the well-defined property of logical strength is neither a virtue nor a vice, so that logically weaker theories are not—all other things being equal—worse or better theories than logically stronger ones. The second thesis is that logical strength does not entail the looser characteristic of scientific strength, and the third is that many modern logics are on a par—or can be made to be on a par—with respect to scientific strength.

0 Introduction

In recent years there has been renewed interest in a broadly abductive approach to the epistemology of logic. The details vary but the central idea is that rival logics are different theories of the relation of logical consequence, and the best theory is the one which is adequate to the data and possesses the most theoretical virtues—perhaps simplicity, strength,

*I'm grateful for helpful discussion from the commentators and audience members at the 2017 Rutgers Epistemology Workshop, a Thursday Seminar at Australia National University, a presentation at the 2017 Australasian Association of Philosophy conference in Adelaide, and discussion at Iowa State University in September 2017. Thank you in particular to Christian Barry, JC Beall, Stephen Biggs, Geoff Brennan, Alex Bryne, Dave Chalmers, Janice Dowell, Alvin Goldman, Dan Marshall, David M. Miller, Paul Oppenheimer, Graham Priest, Dave Ripley, David Sobel, Alex Sandgren, Nic Southwood, Katie Steele, Daniel Stoljar, Una Stojnić, James Willoughby. I'm also grateful for the time and discussion allowed to me by a two month visiting fellowship at the ANU.

1Priest (2006a); Russell (2014); Hjortland (2017); Beall (2017); Williamson (2017)
elegance, unity, symmetry, or ontological parsimony—and least theoretical vices—such as ad hocery, inelegance, or ontological profligacy.

As Hjortland (2017) has noted, such agreement on the methodology for determining the correct logic has not resulted in agreement about the correct logic. Priest (2006b) holds that the correct logic is paraconsistent, Williamson (2017) that it is classical, Beall (2017) argues for the Logic of First Degree Entailment (FDE), and Hjortland and I are are pluralists (though of different kinds).

Looking further back, Quine (1951) endorsed an abductive methodology and he embraced classical logic but—unlike Williamson—he thought that second-order and modal extensions were illegitimate. Carnap (1950) meanwhile, thought that finding the best logic was a matter of finding the best overall linguistic framework, something that could itself be chosen on the basis of virtues like usefulness and simplicity, and unlike Quine, he embraced the prospect of non-classical logics.

The broad agreement on methodology permits a hope that we might eventually be able to determine which is the correct logic to everyone’s satisfaction. But the widespread disagreement about the results suggests something that was perhaps independently plausible anyway: the abductive method in logic is not yet in very clear focus. If Beall can maintain that it leads to FDE, and Williamson that it results in classical logic—even while they both agree on the content of those two logical theories—it suggests a lack of agreement about what the vices and virtues amount to, which theories possess them, and likely also what data adequacy looks like.

It would thus be helpful to clarify the abductive method in logic, with a view to making its consequences more apparent. The present paper focuses on one detail of this project: the putative theoretical virtue of strength.

1 Strength

Strength has often been thought to be a virtue in empirical scientific theories and it is frequently assumed that logical strength is a virtue in a logic. Williamson writes:

2Higher-order modal classical logic, according to Williamson (2013)

3Pluralism might seem at odds with the idea that the epistemology of logic works by inference to the best explanation, but the tension here can turn out to be superficial: many varieties of logical pluralism arise from thinking that validity is relative to something (e.g. cases for Beall and Restall (2006), sets of logical constants for Varzi (2002), and kinds of truth-bearer for Russell (2008)) and so the plurality of theories endorsed are often endorsed as theories of different, though related, things, so that there is no difficulty with assuming that one theory might be the best for one, and another for the other.

4(Quine, 1986, 85–86). Quine was not averse to the idea that classical might be given up in response to new data, such as that from quantum mechanics. However he seems to have regarded the costs of such a move as very high.

5"The first attempts to cast the ship of logic off from the terra firma of the classical forms were certainly bold ones, considered from the historical point of view. But they were hampered by the striving after ‘correctness’. Now, however, the impediment has been overcome, and before us lies the boundless ocean of unlimited possibilities.” (Carnap, 1937, xv) Haack (1978) might also be thought to display sympathy with the abductive approach.
“Once we assess logics abductively, it is obvious that classical logic has a head start on its rivals, none of which can match its combination of simplicity and strength. Its strength is particularly clear in propositional logic, since PC is Post-complete, in the sense that the only consequence relation properly extending the classical one is trivial (everything follows from anything). [...] In many cases, it is unclear what abductive gains are supposed to compensate us for the loss of strength involved in the proposed restriction of classical logic.” (Williamson, 2017, p.19)

Some supporters of weaker logics have agreed that logical strength is a virtue, though unlike Williamson they hold that it is one they are obliged to forgo for some other benefit:

“Weakening classical logic [...] is not something to be done lightly. There are some obvious advantages to keeping classical logic even for “circular” predicates: advantages of simplicity, familiarity, and so on. Choosing to forgo these advantages has its costs. But I will argue [...] that the disadvantages of keeping classical logic for “circular” predicates are also very great, so that the undoubted cost of weakening the logic is worth bearing.” (Field, 2008, 15)

This concessive view is common and it is consistent with the idea that logical strength is a genuine virtue. A more radical view—sometimes hinted at recently in Shapiro (2014), Hjortland (2017), and Beall (2017)—would be that logical strength is really a theoretical vice, and equivalently, that logical weakness is a theoretical virtue, perhaps because weaker logics allow for the development of a broader or better range of extra-logical theories, or allow us to distinguish more possibilities (e.g. distinguish the possibility that \(\neg\neg P\) is true from the possibility that \(P\) is true.) Summing up such radicals, Williamson writes:

“One often encounters various forms of exceptionalism about logic, according to which weakness is a strength in logic, because weak logics leave open more possibilities, prejudge fewer issues, and achieve higher levels of neutrality.” (Williamson, 2017, 18)

Contrary to all three of these views, the first part of the present paper will argue that that logical strength is neither a virtue nor a vice in a logical theory. However, this thesis about logical strength is in tension with a claim made by Williamson (2017) according to which logical strength entails the putative virtue of scientific strength. So I also argue that logical strength does not entail scientific strength after all, and that in fact many of the logics in contention are on a par with respect to scientific strength, in spite of their differences in logical strength.

The next section lays some groundwork and attempts to ward off misunderstanding by distinguishing abductivism from its close companions:

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6The reference to Post-completeness here makes it clear that logical strength is one of the flavours of virtuous strength that Williamson has in mind.
anti-exceptionalism and Quineanism about logic. Section 3.1 then focuses on logical strength and section 3.2 on scientific strength.

2 Abductivism and what it is not

The heart of the abductivist approach consists in two claims. The first is holism about the justification of logic; it is entire logics—rather than isolated claims of consequence—that are justified (or not). The second is that what justifies a theory is adequacy to the data, and the possession of virtues and absence of vices.

Abductivism has been associated historically with a broadly anti-exceptionalist view of logic. (Quine, 1951, 1986; Hjortland, 2017) Exceptionalism about logic is the view that logic is different from the empirical sciences, perhaps by being a priori, analytic, necessary, normative, intuitive, basic, about language (or perhaps topic neutral), and argumentatively neutral between the two sides of any debate. By contrast anti-exceptionalists deny—to varying degrees—that logic has the properties attributed by exceptionalists. For example, Harman (1986) argues that logic is not normative, Williamson that it is not epistemically analytic (Williamson, 2008, Ch4), not metalinguistic (Williamson, 2013, p.93), and not argumentatively neutral (Williamson, 2012), Hjortland (2017) holds that it is not a priori, and most famously, Quine (1951) argued that the epistemology of logic was broadly abductive and as a result neither analytic, nor apriori, nor necessary.\(^7\)

Given this history, it is important for me to stress that abductivism is independent of many other anti-exceptionalist views. One might hold, for example, that the epistemology of logic is abductive, without thinking that logic is a posteriori.\(^9\) If one thinks that the evidence for a logic is itself a priori and that the vices and virtues on which a theory should be judged are discoverable a priori—perhaps elegance and simplicity are like this—then one can be both an abductivist and an a priorist about logic. On the other hand, if one thinks that logical theories should be assessed alongside theories in physics, and the resulting pairings assessed for empirical adequacy and virtue (e.g. quantum mechanics plus classical logic vs quantum mechanics plus quantum logic) then logic’s justification will be a posteriori. Abductivism alone will not decide the a priori/a posteriori question.

If we are concerned to make abductivism attractive to contemporary logicians, then perhaps the most important independence to note is that—contra Quine—one can be an abductivist without holding that logic is contingent. At the end of “Two Dogmas” Quine’s working definition of analyticity was “statement that is true come what may”, i.e. something which he assumed meant the statement could not rationally be revised.\(^8\)

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\(^7\)Quine (1951)

\(^8\)Anti-exceptionalism comes in degrees, and (perhaps confusingly) many anti-exceptionalists deny that logic is exceptional in one way, while agreeing that it is exceptional in others. They might still be regarded as anti-exceptionalists, perhaps because the theses they need to do the most work arguing for are anti-exceptionalist.

\(^9\)See e.g. Biggs and Wilson (2017).
He and many of his readers also thought that analyticity was the only hope we had of accounting for necessity. So if “analytic” meant true come what may, and logic turned out to be revisable (and hence not true come what may), logic could not be analytic, and hence could not be necessary either.

Today, much has changed. Many philosophers think that statements can express necessary truths without being analytic. Elsewhere I have argued that analyticity is a matter of truth in virtue of meaning, a property which depends on meaning, and that the epistemology of meaning is also abductivist—so that theories of meaning, and their attendant results for analyticity, are themselves rationally revisable. Moreover, aside from this view of analyticity, we can recognise that the epistemology of necessary truths can allow that we rationally revise our beliefs about which claims are true. For example, *Hesperus is not Mars* is an a posteriori necessary truth whose justification depends on the place of Mars and Venus in our current theory of the solar system. An epistemically isolated community just beginning to study the skies might initially adopt a theory according to which Hesperus is not Mars, later conjecture that Hesperus might actually be Mars in response to a new theory about why Hesperus might appear red under certain conditions, confirm this theory with (somewhat misleading or partial) data, and eventually give it up again once it is realised that the two are sometimes visible simultaneously. But this rational revisability of claims about the identity of Hesperus and Mars is compatible with the fact that *Hesperus is not Mars* expresses a necessary truth. An abductive methodology for a certain domain is compatible with some or more of the truths of the domain being necessary.

If abductively justified truths can be necessary, apriori and even true in virtue of meaning, one might wonder what abductivism rules out. One view that is incompatible with abductivism is a view on which individual claims about entailment are justified atomistically, rather than in the context of a whole theory. Let an *E*-sentence be an atomic sentence in which ‘⊨’ is the main predicate, as it is in these two:

\[
A, A \rightarrow B \vdash B \\

\vdash \neg(A \land \neg A)
\]

And let an E-literal be either an E-sentence or its negation, as in:

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10 Russell (2012b)
11 Russell (2014)
12 A distracting complication: by the necessity of logic I mean the fact that the instances of many logical truths must express necessary truths (such as “Snow is white or it is not the case that snow is white”) and also the fact that if a set of sentences Γ entails a sentence A then there is no possible world where every member of Γ is true and A false. This necessity is assumed to be a feature of many logics, and this is brought out when we add □ to the language—representing metaphysical necessity—and endorse the familiar principle of *necessitation*—a valid rule in all normal modal logics (including, say S4 and S5)—which allows us to ‘Box’ any logical truth: \( \vdash A \equiv \square A \). That said, there are some important and interesting logics some of whose logical truths are contingent, e.g. in Kaplan (1989). See also Russell (2012a) for further discussion.
13 the ‘E’ is for entailment.
Then epistemic atomism about logic holds that E-literals are justified E-literal-by-E-literal. This might be endorsed by someone who held e.g. i) that E-literals are justified by proving them from antecedently established E-literals, and that the most basic E-literals are a priori intuited to be correct or ii) that E-literals are established on the basis of model-theoretic proof or counterexample, and that it is obvious which model theory is the best or iii) that E-sentences are established by formal, natural deduction proofs, whose basic rules are valid-in-virtue-of-meaning and easily recognisable as such by anyone who speaks the language.

What is distinctive about such epistemically atomist views is that individual E-literals are established or rejected on their own merits. And if this is right then the best theory could just be the set of all the individual sentences which have been established (and perhaps the negation of the disjunction of all the ones which have been refuted.) This would not be an abductivist view.

Still, I should note that even an abductivist will think that there is a place for atomistic proofs of E-sentences and counterexamples to negative E-literals; once we have justified our logic using abductive means, we will be able to use (or at worst develop) a model theory and a proof theory for that logic, and these can then be used to give proofs of individual E-sentences, and counterexamples to not-E-sentences.

Finally, one anti-exceptionalist view that I do think is supported by abductivism is the view that logics are rationally revisable, either through of the discovery of new data (perhaps concerning things like vague predicates, quantifiers or various paradoxes) or because the best available logical theory at one time can later be superseded by another, new and more virtuous theory, as one might think happened with Frege’s theory of polyadic quantification, or with Kripke’s model theory for modal logics.

3 Strength

The putative vices and virtues for logical theories fall into two kinds. If a characteristic is of the first kind, it is precise and well-defined, and it is uncontroversial whether a theory has it. This group includes completeness, compactness, decidability, and expressive power. Still, it is frequently controversial whether this kind of characteristic qualifies as a virtue or vice. For example, it is uncontroversial whether first-order classical logic is decidable and complete (it isn’t decidable, it is complete), but controversial what these features mean for the attractiveness of the theory of first-order classical logic. Characteristics in the second group are harder to define. These include simplicity, inelegance, symmetry, unity, ad hocery, etc. Here the status as a virtue or vice is rarely questioned, but the presence of the virtue in any one theory is disputed.

One reason strength is a promising starting place is that it can seem to have the positive feature of both groups: one might think both that logics fall into clear, uncontroversial strength hierarchies and also that
strength is uncontroversially a virtue. Unfortunately, this early promise will dissolve under scrutiny.

### 3.1 Logical Strength

Logicians are used to talking about a certain kind of strength. Limiting ourselves to sentential logics for a moment, classical logic is stronger than the paraconsistent logic LP and stronger than the strong Kleene logic K3. LP and K3 are each stronger than FDE, which is in turn stronger than the empty logic—call that NL for the nihilist logic. And at the other end, the fact that classical logic is Post-complete means that the only logic strictly stronger than it is Triv—the trivial or universal logic on which every sentence $A$ follows from every set of premises $\Gamma$.$^{14}$

Expanding our set of logical constants produces another strength hierarchy; first-order logic is stronger than sentential logic, second-order logics are stronger than first-order logics, sentential modal logics are stronger than non-modal sentential logics. Of course, sentential modal logics form their own well-known hierarchy as well.

In each of these three diagrams, the arrows represent the same relation, ‘is stronger than’ and I will call this kind of strength \textit{logical}:$^{15}$

\begin{definition}[Logical Strength (logics)] Logic $F$ is \textit{logically stronger} than logic $G$ just in case for all sets of sentences, $\Gamma$, and sentences, $A$,

\[
\text{if } \Gamma \vdash_G A \text{ then } \Gamma \vdash_F A
\]
\end{definition}

$^{14}$There are non-trivial logics which have entailments that classical logic does not, but these must always lack some other entailment that classical logic is committed to; connexive logics, for example, may contain the non-classical logical truth $\models (A \rightarrow B) \rightarrow (\neg A \rightarrow \neg B)$ but they all lack the classical $\models \neg A \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow \neg B)]$. (Priest, 2006a, p. 156)(Wansing, 2016)

$^{15}$This is Williamson’s terminology from Williamson (2013). Hjortland (2017) uses “deductive strength” to talk about the same relation.
but not vice versa (that is, there is some \( \Gamma, A \) such that \( \Gamma \models F \) \( A \) but \( \Gamma \not\models G \) \( A \).)

If we limit our attention to the positive aspect of logics (that is, their sets of positive E-sentences, ignoring the negated E-sentences, as well as any other parts of the theory, such as definitions of models or truth) this notion is a special case of a more general notion of logical strength:

**Definition 2 (Logical Strength (theories))** Theory \( F \) is stronger than theory \( G \) just in case every sentence in theory \( F \) is in theory \( G \), but not vice versa.

That is, we get the first definition from the second by thinking of logics as sets of sentences of the form \( \Gamma \models A \).\(^{16}\)

If logical strength were a theoretical virtue—or if it were a theoretical vice—then these rankings could be useful in choosing the correct logic. It is often thought that strength is a virtue in scientific theories, and it is sometimes thought that logical strength is a virtue in logical theories. Perhaps even more often, it is taken to be obvious that too much logical weakness would be bad. Non-classical sentential logics always seem to be in danger of being too weak. Too weak for what? Perhaps too weak to do metatheory, or too weak for closing theories. Too weak for mathematical proof, too weak to give a foundation for arithmetic, or too weak to do exciting metaphysics. In sum, one might worry that if the correct logic is very weak, then logic itself might not be very useful, even that logic as a discipline might not be very interesting. In any event, defenders of logics that have been weakened in one way have often been quite keen to prevent them from being further weakened in some other way.

Recently though, some logicians have argued that weaker logics can be more useful than stronger ones.\(^{17}\) One reason given is that weaker logics allow for the development and study of a greater variety of theories, such as theories of truth, arithmetic, or set theory. If we accept classical logic, then on pain of triviality via the Liar paradox, we are obliged to reject a theory of truth which includes commitment to every instance of the disquotational schema:

\[
\text{True}(\langle A \rangle) \text{ if and only if } A
\]

Weaker logics can avoid the derivation of a contradiction (hence avoiding the paradox), or avoid trivialisation once the contradiction is derived, and so allow us to accept and study this rather elegant theory of truth. Chapter 3 of (Shapiro, 2014) also gives examples of mathematical theories, such as Heyting arithmetic, which are developed in intuitionistic logic and would be trivialised by the addition of the law of excluded middle. A second reason weaker logics are sometimes favoured is that they allow us

\(^{16}\)Some logicians are likely to want a more general conception of a logic still, perhaps allowing multiple conclusions and otherwise more complex sentences than these. The logics mentioned in the present paper, however, are well-characterised by the set of sentences of the form \( \Gamma \models A \) they license relative to a language, so I will stick with this characterisation for simplicity, with the caveat that things might get more complicated e.g. when we are talking about substructural logics. Section 3 of Hjortland (2017) is helpful on this topic.

\(^{17}\)(Shapiro, 2014, Ch.3), (Beall, 2017) (Hjortland, 2017)
to draw more distinctions, such as the distinction between a situation in
which $\neg \neg P$ is true, and one in which $P$ is true.\(^{18}\)

Another important feature of a logical system is its expressive
power or discriminatory power. Logics differ in what they
can talk about. Some logics can characterize structures that
other logics cannot. \([\ldots]\) Although expressive strength can
come with a cost (e.g., deductive limitations), it can clearly
also be an advantage, not least because expressive strength
may improve explanatory strength. A language capable of finer
discriminations will prove superior in explaining finer-grained
phenomena. (Hjortland, 2017, p.646)

There are really two versions of the pro-weakness view: a concessive
version, and a radical one. The concessive view concedes that strength is a
virtue, but adds that we’re forced to give it up in order to embrace our best
extra-logical theories or increase discriminatory power. The radical view
says that strength is a vice and weakness is the corresponding virtue.\(^{19,20}\)

So we have three views: on the first, logical strength is a virtue, on
the second, it remains a virtue but is outweighed, an on the third, logical
strength is a vice. Now I want to argue that all three of these views are
wrong; logical strength is neither a virtue nor a vice.

I take as a premise in my argument that if a characteristic is a theo-
retical virtue (vice), then all things being equal, if theory F has more of
that characteristic than theory G, then theory F is better (worse) than
theory G.\(^{21}\)

Next, we observe that logical strength is not like this. A theory may
be too strong and not just because its strength forces a reduction in some

\(^{18}\)Humberstone (2005)

\(^{19}\)We might call this the “Sklavenmoral” approach to logical strength.

\(^{20}\)Beall’s published view seems to have a bit of both but I believe he favours the radical
approach and he likes to rebrand weakness as the much more attractive sounding “depth”: “Logic, on this ‘deeper’ picture, still affords a natural treatment of the paradoxes. The ‘solutions’ afforded by standard (though lopsided) subclassical logics carry over to FDE. The logic is weak (or ‘deep’) enough to accommodate standard paradoxical notions (e.g., truth, exemplification, etc.) By diving deeper than the standard lopsided subclassical levels we do not lose the options for naturally resolving paradoxes; we have more options—treating some of them as ‘gappy phenomena’ and some ‘glutty phenomena’ versus trying to squeeze them all into one category or the other, regardless of how unnatural the fit appears.” (Beall, 2017, 13)

\(^{21}\)Commenters at the 2017 Rutgers Epistemology Workshop and the 2017 AAP in Adelaide
have pointed out to me that Aristotle’s conception of the personal virtues is not like this: patience is a virtue, but it is not true that the more patient someone is, the better. There is such a thing as too little patience, and but also such a thing as too much. Similarly with
courage, modesty, temperance and the other personal virtues. I take this point, but I also
think that the theoretical virtues, with which this paper is concerned, are different. Elegance,
for example, is not a level that a theory has to match, nor is simplicity; assuming both theories
are equal in other ways, additional elegance always better, and additional ad hocery always
worse.

\(^{22}\)Here the “all things being equal” is intended to rule out cases where F is better than
G because of a change in vices and virtues that are distinct from the characteristic under
consideration, as it might be if F possessed both more logical strength and more simplicity.
Then F would be better but not because of the increase in logical strength.
other virtue, but because the additional strength is itself a problem. The same goes for logical weakness. It is in principle possible for a logic to be too weak, so that another, strictly stronger logic would be an improvement precisely because it is stronger (and not merely because the stronger logic improves in other respects, such as simplicity.)

To take an example, our strongest sentential logic is Triv, which is the logic on which any arbitrary conclusion follows from any set of premises at all. Triv, it is widely agreed, is not our best logic, and there are two obvious problems with it: i) it says that arguments are valid when they are not, and ii) it is too strong. But on consideration, we can see that these are the same problem: Triv is too strong, and what it is for a logic to be too strong is for it to say that arguments are valid when they are not. Thus the key problem with Triv is that it has too much logical strength. This could not be the case if logical strength were a virtue.

An analogous example with Ni will demonstrate that logical strength is not a vice either. Rather, getting the best level of logical strength is not a matter of getting as much as we can, but of hitting a target: not too strong, not too weak. We can characterise this target precisely and uncontroversially, though not in a way that provides a neutral arbiter between rival logics: a logic $F$ has the correct level of strength iff for any set of sentences, $\Gamma$, and sentence $A$, $\Gamma \vdash F A$ if and only if $\Gamma \vdash A$. *Snow is white* does not entail *grass is purple*, so Triv is too strong. On the assumption that *snow is white* and *grass is green* do entail *snow is white*, Ni is too weak. In the middle, the entailment facts become more controversial, which makes things more difficult for us epistemically and dialectically. A paraconsistent logician will claim that classical logic is too strong because it contains $A, \neg A \vdash B$ and a classical logician will say that LP is too weak because it fails to contain the same E-sentence. But our difficulties in settling disputes over the answers to these questions don’t change the fact that logic strength is something that a logic is supposed to get right, rather than something it is always good to have more of.\textsuperscript{23}

It is with some regret that I reach this conclusion. Logical strength would have made a nice theoretical virtue because unlike other characteristics, such as simplicity and elegance, it can be straightforward to see whether one logic is stronger than another. But since logical strength is not a virtue, the clarity of its rankings presents a certain danger. University professors sometimes complain that numerical student evaluations do not measure teaching effectiveness and worse, that in the absence of a better measure, these bad measures get used because administrators can find nothing better. In the absence of a good grip on the genuine theoretical virtues, logical strength presents a similar temptation.

\textsuperscript{23}In his question at the 2017 Australasian Association of Philosophy conference in Adelaide, Dan Marshall suggested an illuminating analogy: saying that logically stronger logics are always better would be like attempting to give a theory of love (i.e. a theory of the extension of the ‘x loves y’-relational predicate) and insisting that theories on which more people love each other are always better than theories on which fewer people love each other.
3.2 Scientific Strength

Logical strength is not the only relevant sense of “strength”. Williamson identifies a different characteristic of theories which he calls scientific strength. This characteristic is harder to pin down (Williamson calls it a “looser” sense of strength) but the core idea is that scientifically stronger theories are more informative and specific. In his main illustration “The time is between 3.14 and 3.16” is scientifically stronger than “The time is between 4.00 and 12.00.”

He suggests that logical strength entails scientific strength, but not vice versa:

“If T is stronger than T∗ in the strict logical sense, then T is also stronger than T∗ in the looser scientific sense, but the converse fails. Both senses are applicable to logical theories. For instance, let PC be standard classical propositional logic, and IC be intuitionist propositional logic. Then every theorem of IC is a theorem of PC but not conversely, since P ∨ ¬P is a theorem of PC but not of IC; likewise for the corresponding consequence relations. Thus PC is stronger than IC in the strict logical sense, and so also in the looser scientific sense.”

I think it likely that Williamson is right that scientific strength is a virtue. But if so, then an entailment between logical and scientific strength threatens my conclusion from the previous section. For suppose scientific strength is a virtue. Then by my own standards, a theory with more of it is ceteris paribus better. But then if L1 is logically stronger than L2, and logically stronger entails scientifically stronger, then L1 must be scientifically stronger than L2 as well. So the fact that L1 is logically stronger than L2 means that L1 is better than L2 after all.

In this section I am going to argue for two additional theses. First, that logical strength does not entail scientific strength; that will dissolve the threat to the conclusion of the previous section. And second, I will argue that there are two senses of scientific strength that one might think relevant to logics. On the first, the logics in the set {Ni, FDE, LP, K3, CL, Triv} are all equal in scientific strength. And on the second, although logically weaker logics are often less scientifically strong than logically stronger logics, any of the logics in our set can be strengthened—without damage to the logic—so that they are as scientifically strong as our logically strongest logic, Triv.

To begin, recall that for one theory to be scientifically stronger than another is to say (loosely) that it is more informative and more precise. But now something about the idea that one contemporary logic is more informative and more precise than another contemporary logic might strike us as odd; none of the logics in our set say that the validity of modus ponens is between 3 and 7, or that the principle of non-contradiction is kinda logically truthy. Modern logic is mathematical, and logics are formulated so that they are determinate, in the sense that for any set of premises, Γ,
and conclusion, \( A \), in the language on which the logic is defined, they say whether or not \( \Gamma \vdash A \).

A common way to introduce a logic, for example, is to first specify the language on which it is to be defined, then define a set of models \( U \) for that language, define the conditions under which a sentence will be true in a model in \( U \), and finally use these things to give the conditions under which a set of premises is counted as entailing a conclusion, e.g. \( \Gamma \models A \) if and only if there is no model \( M \in U \) such that \( M \) makes every member of \( \Gamma \) true and does not make \( A \) true. Whether a model is a member of \( U \) is determinate, and (at least in the logics in our set, and in also in many others) whether a model makes a sentence true is determinate, so the model decides for each argument and argument form, either \( \Gamma \models A \) or \( \Gamma \not\models A \). How could the logic be any more informative or specific than that?

Still, there is a way in which scientific weakness tends to creep into logical theories, and it is a tendency to which weak logics are especially vulnerable.

### 3.2.1 The Vagaries of Classical Recapture

E-sentences are very general claims. For example, according to classical logic, modus ponens—\( A \rightarrow B, A \models B \)—is valid, and if it is, then whatever sentences form the antecedent and the consequent of the conditional, you still can’t have a situation where the conditional and its antecedent are true, but the consequent is not. If modus ponens is valid, it works every time, for every value of \( A \) and \( B \). Similarly for logical truths: if you accept the law of excluded middle then you are committed to \( A \lor \neg A \) being true for all of its instances.

This generality means that non-classical logicians sometimes reject an entire logical principle because of some quite esoteric cases. For example, certain intuitionists think that the law of excluded middle is not valid because of sentences concerning mathematical objects (such as far out members in infinite collections) which have not yet been constructed by us. But this exception is quite specific—it concerns certain sorts of mathematical object—and an intuitionist need not think there are any problems with the law when it is restricted to sentences that speak only of the first 10 natural numbers, or the chairs and people in this room. They need have no quarrel with \( 3 \text{ is prime} \lor \neg 3 \text{ is prime} \), for example, or \( \text{That chair is taken or not taken} \). These things are true and you may safely use them; they’re just not logically true (according to the intuitionist) because for that to be the case they would need to be instances of a law that held quite generally.

Similarly, one kind of dialetheist rejects explosion because they hold that the Liar Paradox shows that some sentences can be both truth and false. This, they think (along with the LP interpretation of the connectives) generates counterexamples to the rule \( A, \neg A \models B \) (see figure 2).

Still, a dialetheist can hold that sentences which don’t contain truth predicates are bound to be true or false (and not both), and so maintain that we make no mistake in applying modus ponens in arithmetic or even to \( \text{if snow is white then grass is green and snow is white} \); in these cases
the move won’t take you from truth to non-truth. But again, this won’t
mean that the argument is logically valid, because that would require it
to be of a form that had no counterexamples anywhere.

Given this, when responding to concerns about their logic being too
weak, a non-classical logician might engage in a project of classical recapture.
They can say, essentially, “yes, my logic is a bit weak, but in practice
you can still use the stronger classical principles quite a bit. Maybe even
to do your metatheory or your arithmetic.” For example, here’s Hartry
Field:

“...we ought to seriously consider restricting classical logic
to deal with all these paradoxes. [...] I say ‘restricting’ rather
than ‘abandoning’, because there is a wide range of circum-
stances in which classical logic works fine. Indeed, I take excluded
middle to be clearly suspect only for certain sentences
that have a kind of “inherent circularity” because they contain
predicates like ‘true’; and most sentences with those predicates
can be argued to satisfy excluded middle too. The idea is not
that we need two logics, classical logic and a paracomplete logic
(one without excluded middle.) On the contrary, the idea is
that we can take the paracomplete logic to be our single all-
purpose logic. But we can recognise the truth of all instances
of excluded middle in certain domains (e.g. those that don’t
contain ‘true’, ‘true of’, ‘instantiates’, or other suspect terms).”
(Field, 2008, p.15)

At the extreme of this strategy there is logical nihilism, which endorses
Ni as the correct logic.26 What makes nihilism remotely tenable is the
idea that while no putative logical entailment holds quite generally (and
in particular, no matter how rich the formal language is) many familiar
logical principles have no counterexamples in familiar languages—that
is, you can accept lots of regular common-or-garden instances of modus
ponens without accepting modus ponens as a logical law.

Still, the project of classical recapture raises worries about the infor-
mativeness and the precision of weaker logics. If our question is ‘which

\begin{tabular}{c|c|c|c|c}
A & B & A & ¬A & B \\
\hline
T & T & T & F & T \\
T & F & T & F & F \\
T & B & T & F & B \\
F & T & F & T & T \\
F & F & F & T & F \\
F & B & F & T & B \\
B & T & B & B & T \\
B & F & B & B & F \\
B & B & B & B & B \\
\end{tabular}

Figure 2: A dialetheic counterexample to explosion

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26Russell (2017)
instances of LL can we use?’ (where LL is some disputed logical law) then the logically stronger logic tells us ‘all of them’ whereas the weaker logic says ‘not all of them’—and this tells us nothing further about which particular instances are untarnished. “Not all” is less informative than “all”. And while in their accompanying remarks logicians sometimes say more about when an instance of a principle which has counterexamples may be used, here they are frequently less careful and precise here than they are when pursuing logic proper.

In the next section I try to clarify and illustrate these ideas with the help of a simple analogy.

3.2.2 Large Squares and Small Squares

Suppose we have a 10X10 grid of Large Squares, numbered 1–100. Large Squares have two states—black or grey. Each Large Square in the 10X10 grid is itself a 10X10 grid, subdivided into 100 Small Squares. Small squares too may be black or grey. If the 100 Small Squares that make up a Large Square are all black, then the Large Square is itself black. But if any of the Small Squares that make up a Large Square are grey, then the Large Square is grey.

A Large Square Theory is a theory which gives information about the colour of Large Squares. A Large Square Complete Theory says, for each of the Large Squares 1–100, whether that square is black or grey. (You might think of a it as a complete function whose domain is the set of numbers 1–100 and whose range is the set of the colours black and grey.) When we restrict our attention to Large Squares, every Complete Large Square Theory is as scientifically strong as every other Complete Large Square Theory. There are two states for each Large Square, and if a theory specifies exactly one of those states for each square, its work is done.

This is the situation with respect to rival logics defined on the same language. The job of the logical theory is to say for each argument form (though here there are more than 100) whether it is logically valid (black) or not (grey.) Triv, Classical logic, LP, K3, FDE, and Ni do this for every argument form, and hence they are on a par with respect to scientific strength. We might express this by calling them Large Square Complete.

A scientifically weaker Large Square Theory would be one that was less informative—perhaps by only telling us about some but not all of the large squares—or less precise—perhaps by speaking of ranges in which black squares could be found. A scientifically weaker logic would be one which was less informative—perhaps by only telling us about the valid and invalid argument forms containing conditionals—or one which was less precise—perhaps by speaking of the valid arguments in a way that doesn’t allow us to determine whether particular arguments are valid. Candidates might include a theory L3, which says that there are at least 8 valid argument forms (but not which ones they are) or L4, which says that about half of all argument forms are valid. It’s obvious that none of the logics in our set is flawed in this way.

Since some of our logics are logically stronger than others, and they are (in the Large Square sense) on a par with respect to scientific strength, it follows that logical strength does not entail scientific strength (on this
sense of scientific strength.) Still, when thinking about classical recapture we noted that there is a way in which classical logic is more informative, and hence scientifically stronger (in a different sense.) We can develop the analogy to illustrate this sense of scientific strength too.

We might wish to be informed about the colours of the Small Squares. Call a theory Small Square Complete if it tells us the colour of every one of the 10 000 Small Squares (think of a Small Square Complete theory as a complete function from pairs of numbers between 1 and 100 to the colours black and grey; the first number gives the Large Square of which the Small Square is a part, and the second number specifies its position in that Large Square.)

There is only one Large Square Complete Theory which determines a Small Square Complete Theory: the one which tells us that every Large Square is black. Among the other Large Square theories, some are more informative about Small Squares than others. The Large Square theory which says that the first 50 Large Squares are black and the rest grey (call that LS-50) tells us the colours of more Small Squares than the Large Square theory which says that the first 40 Large Squares are black and the rest grey (call that LS-40.) Where the two theories differ, LS-50 says that a Large Square is black and LS-40 that it is grey, and so it follows from LS-50 that each of the Small Squares in the disputed Large Square are black, whereas from LS-40 it follows only that at least one of the Small Squares is grey. LS-50 tells us more (about the colour of Small Squares) and is thus scientifically stronger (in the Small Square sense) than LS-50.

Returning to the logics, Classical Logic and K3 (for example) are both, as it were, Large Square Complete—they tell us whether or not each argument form is logically valid. But we might wish to be informed about more than just argument forms and validity. We might also want to know about all the instances of the forms. Now strictly, instances of argument forms are logically valid if and only if they are instances of some logically valid argument form. (Dialetheists don’t say some instances of explosion are logically valid; logical validity is very general, and so if there is a counterexample anywhere it fails.) But suppose there is a property of instances of argument forms that may be possessed by one instance and not another instance of the same argument form, and this property is such that if every instance of an argument form has it, then the resulting argument form is logically valid, but if a single instance of the argument form lacks it, then the form is not valid. Call that property subvalidity.

LP says that some argument forms are logically valid. CL is logically stronger, which means that it says that all those LP argument forms are valid, plus some more. Since the logical validity of a form entails that every instance of the form is subvalid, CL tells us that all the instances

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27Henceforth I’ll assume that all the Large Square Theories under consideration are Large Square complete.

28One’s theory of sub-validity is will vary with one’s view of logical consequence. For the Tarski of (Tarski, 1936) and (Etchemendy, 1999) it would have been having a true conclusion if you have all true premises (where the if is an ordinary material conditional.) But we could also think of it more substantially as being necessary truth-preservation, or being truth-preserving in virtue of meaning. What subvalidity amounts to is an important and interesting question, but I won’t defend a particular approach here.
of the disputed forms are subvalid, while LP tells us that at least one of the instances of the disputed forms is not subvalid. LP alone does not tell us, for each of the disputed forms, which instances are subvalid and which not. Thus, with respect to subvalidity and argument instances, the logically stronger CL is more informative than the logically weaker LP. That is, the logically stronger logic is scientifically stronger than the logically weaker one, once we broaden our sense of informativeness to include information about subvalidity in this way.

One might be tempted to defend weaker logics on the grounds that we shouldn’t expect them to tell us about things other than what they are a theory of—it is no criticism of a theory of music that it doesn’t tell us as much as it might about elephants. Similarly, LP is a theory of logical validity, not subvalidity (this line would run) and so its scientific strength with respect to facts about subvalidity is irrelevant. But this defence seems forced. Surely the reason we are interested in logical form at all is that we want to know which particular arguments are good. We’ve tended to study logical validity, but the weaker our logic is, the more important subvalidity seems. If modus ponens and explosion aren’t logically valid but some instances are subvalid, we would like to know which ones.

One might then think, that this means that there is a sense of scientific strength on which it is entailed by logical strength, but this would be too fast. That is because every Large Square Theory which is not already Small Square Complete can be extended to a theory which is Small Square Complete, with no change to what the theory says about Large Squares (so essentially, without changing the Large Square theory itself.) We simply specify the colours of the remaining Small Squares in a way that doesn’t make any of the grey Large Squares black.

The same goes for logics. The only sentential logic which is “Small Square Complete” is Triv. All the others, including CL, say of the invalid argument forms only that there is at least one instance which is not subvalid. Each logic except Triv can therefore engage in a project of “Triv recapture” which specifies which instances of the argument forms it calls “logically invalid” are subvalid. As long as it does not say that all the instances of one of those forms are subvalid, the logic itself remains unchanged. Hence, even on the broader sense of Scientific Strength, on which we pay attention to argument instances and subvalidity (in addition to argument forms and logical validity) logical strength does not entail scientific strength, because scientific strength (how close we are to Small Square Completeness) can vary independently of logical strength. LP is logically weaker than CL, but they can be made on a par with respect to Scientific Strength through a process of precise and informative Triv recapture.

Classical logicians might suspect that this process of recapture will not be pretty. I think their concern is fair. Once K3 and FDE (for example) have been extended to theories which are Small Square Complete, the resulting theory+extension pairs will still need to be assessed for data-adequacy and theoretical virtues and vices like simplicity and unification. If the resulting accounts are inelegant and ad hoc, that will be argument against them. But I want to urge two reasons for caution here. First, if it is argument instances and their subvalidity that we want to be informed
about, then all theories except Triv require extensions. We shouldn’t be comparing LP plus its extension to CL, but to CL plus its extension. Second, the claim that one of these logic-extension pairs will be more virtuous than another amounts to a predication about theories not yet developed.

4 Conclusion

This paper has looked at the characteristic of strength in assessing logical theories. It has argued that logical strength is neither a vice nor a virtue, on the grounds that logical strength does not have the features we expect vices and virtues to have; making a theory logically stronger will not, all things being equal, make it better.

Section 3.2 then argued that modern rival logics, including those in the set \{Ni, FDE, LP, K3, CL, Triv\}, are frequently on a par with respect to scientific strength if we are interested in being precisely informed about logical validity. Admittedly, once we extend our interests to argument instances and subvalidity there is a sense in which logically stronger logics tend to be scientifically stronger than logically weaker logics. But even here, the scientific weakness is not tied to the logic, since the logic can be supplemented with a theory of subvalidity. Once this is done the logic remains as it was, but the overall theories are on a par with respect to scientific strength. Either way, logical strength does not entail scientific strength.

References


